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A numerical method to determine the two-dimensional, unsteady flow about a flat plate.

Wolcott, Hugh D.

Princeton University

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Hugh D. Wolcott

A NUMERICAL METHOD TO DETERMINE THE TWO-DIMENSIONAL, UNSTEADY FLOW ABOUT A FLAT PLATE

by

Hugh D. Wolcott

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for the degree of Master of Science in Engineering
from Princeton University, 1969.

Signature of Author

ACKNOWLEDGMENT

I feel indebted to the United States Navy and to the Department of Aerospace and Mechanical Sciences, Princeton University, for making this study possible. I am most grateful to Professor George L. Mellor for his guidance and encouragement. I would like to express my gratitude to Dr. H. James Herring, Mr. Thomas F. Balsa, and Mr. Ronald M. C. So, for their advice and suggestions. Finally, I wish to thank Mmes. Esther Olsen and Helen Reaser for typing the manuscript and preparing the figures.

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ABSTRACT

Numerical solutions are obtained for two-dimensional, unsteady potential flow about a flat plate. Two types of flow are investigated: small amplitude oscillation about a spanwise axis and impulsive acceleration of a nonoscillating plate. Pressure, forces and moments, and kinematics of the vortex wake are calculated.

In this work vortex sheet shedding from the trailing edge is included. In future efforts, vortex sheet shedding from trailing and leading edges are contemplated.

NOMENCLATURE

c	-	chord length
D'	-	drag per unit span
$(\underset{\sim}{i}, \underset{\sim}{j}, \underset{\sim}{k})$	-	unit vectors in (x-, y-, z-) directions respectively
L'	-	lift per unit span
M	-	number of time increments taken
N	-	number of evenly distributed bound vortices
$\underset{\sim}{\hat{r}}$	-	position vector
$\hat{s}$	-	arbitrary running variable
t	-	time
T.V.S.	-	trailing vortex sheet
$\underset{\sim}{\hat{U}}_\infty(t)$	-	free stream velocity
$\hat{U}_{\infty_f}$	-	final free stream velocity
$\underset{\sim}{\hat{V}}(\underset{\sim}{\hat{r}}_o)$	-	field velocity at r_o
V.S.	-	vortex sheet
$(\hat{x}, \hat{y}, \hat{z})$	-	absolute reference system
$\hat{\gamma}(\hat{s})$	-	vortex strength distribution
$\hat{\Gamma}$	-	free vortex strength
θ_o	-	amplitude of oscillation
$(\hat{\xi}, \hat{\eta}, \hat{\zeta})$	-	body fixed reference system
$(\underset{\sim}{e}_\xi, \underset{\sim}{e}_\eta, \underset{\sim}{e}_\zeta)$	-	unit vectors in (ξ-, η-, ζ-) directions respectively
$\hat{\omega}$	-	angular frequency

Nondimensional quantities

$c_l = L'/\frac{1}{2}\rho\, \hat{U}_{\infty_f}^2\, c$

$c_d = D'/\frac{1}{2}\rho\, \hat{U}_{\infty_f}^2\, c$

$\underset{\sim}{r} = \hat{\underset{\sim}{r}}/c$

$s = \hat{s}/c$

$(x, y, z) = (\hat{x}, \hat{y}, \hat{z})/c$

$(\xi, \eta, \zeta) = (\hat{\xi}, \hat{\eta}, \hat{\zeta})/c$

$\underset{\sim}{U_\infty}(\tau) = \hat{\underset{\sim}{U_\infty}}(\tau)/\hat{U}_{\infty_f}$

$\omega = \hat{\omega} c/\hat{U}_{\infty_f}$

$\gamma = \hat{\gamma}/\hat{U}_{\infty_f}$

$\Gamma = \hat{\Gamma}/\hat{U}_{\infty_f}\, c$

$\tau = t\, \hat{U}_{\infty_f}/c$

Subscripts

(x, y) - component in $(x-, y-)$ direction

(ξ, η) - component in $(\xi-, \eta-)$ direction

I. INTRODUCTION

Present generation computers make numerical approach to unsteady, two-dimensional subsonic flow attractive. In this study, two types of flow about a flat plate in nonlinear motion are investigated. Although having different objectives, related studies have been carried out by J. P. Giesing [1], N. D. Ham [2], and F. H. Abernathy and R. E. Kronauer [3].

Observations show that unsteady flow about a lifting body is accompanied by a vortex sheet being shed from the trailing edge. In the present study of unsteady flow about a flat plate, the plate is also replaced by a bound vortex sheet. Both the bound and the free vortex sheet are then replaced by a finite number of discrete point vortices; thus, the integral equation of potential theory representing the field velocity is reduced to a finite difference equation. With appropriate application, this finite difference equation is used to determine the positions of the shed free vortices as a function of time and to express the boundary condition at any instant of time. In regard to the boundary condition, it is stipulated that the strengths of the vortices, both bound and shed, are such that the trailing streamline is tangent to the trailing edge. This satisfies the Kutta condition. Together with Kelvin's theorem, it is sufficient to determine uniquely the strengths of all vortices at any given instant of time. Steady flow is treated as a degenerate case of the above.

The kinematics of the shed vortex sheet and strength distribution of the airfoil and free vortices are determined. Calculation of such important physical parameters as stability derivatives follows immediately.

II. THE BASIC EQUATIONS

A potential flow model is used for unsteady flow about an infinite flat plate. Laplace's equation governs the velocity field and pressure can be calculated from Bernoulli's equation. From Kelvin's theorem the change of circulation about the flat plate is balanced by the shed vortex sheet(s)[1]. Finally, it is assumed that the Kutta condition is satisfied; this condition is sufficient to determine the rate of vortex sheet shedding.

Using thin airfoil theory, the flat plate is replaced by a vortex sheet where vortex strength distribution is adjusted to make the flat plate a streamline of the flow.

Consider an arbitrary flow field containing a vortex sheet with vortex strength distribution $\hat{\gamma}(\hat{s})$. Vorticity is considered positive when inducing counterclockwise circulation. $\hat{\underset{\sim}{r}}$ is the position vector

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1. Bluff body flow is stipulated to be accompanied by vortex sheets shed from the leading and trailing edges.

on the sheet while $\hat{s}$ is the running variable along the sheet. The field velocity at $\hat{\underset{\sim}{r}}_o$ is then given by

$$\hat{\underset{\sim}{V}}(\hat{\underset{\sim}{r}}_o, t) = \int_{V.S.} \left\{ \frac{\hat{\gamma}(\hat{s}, t)}{2\pi} \underset{\sim}{k} \times \frac{(\hat{\underset{\sim}{r}}_o - \hat{\underset{\sim}{r}})}{|\hat{\underset{\sim}{r}}_o - \hat{\underset{\sim}{r}}|^2} \right\} ds + \hat{U}_\infty(t)\, \underset{\sim}{i}$$

$$= \int_{V.S.} \left\{ \frac{\hat{\gamma}(\hat{s}, t)}{2\pi} \left[\frac{-\underset{\sim}{i}\,(\hat{y}_o - \hat{y}) + \underset{\sim}{j}\,(\hat{x}_o - \hat{x})}{(\hat{x}_o - \hat{x})^2 + (\hat{y}_o - \hat{y})^2} \right] \right\} ds + \hat{U}_\infty(t)\, \underset{\sim}{i} \tag{1}$$

The same equation written nondimensionally is

$$V(r_o, \tau) = \int_{V.S.} \frac{\gamma(s, \tau)}{2\pi} \left\{ \frac{-\underset{\sim}{i}(y_o - y) + \underset{\sim}{j}(x_o - x)}{(x_o - x)^2 + (y_o - y)^2} \right\} ds + U_\infty(\tau)\, \underset{\sim}{i} \tag{2}$$

Hereafter we will omit specific reference to the temporal argument.

A description of the geometry of the field is appropriate. The absolute reference system is assigned coordinates (x,y,z) with their origin at the geometrical center of the flat plate. In the present study the center of the flat plate is therefore fixed but the plate is free to rotate in an arbitrary manner. $\underset{\sim}{i}$, $\underset{\sim}{j}$, and $\underset{\sim}{k}$ are unit vectors in the x, y, and z directions respectively. A body fixed coordinate system (ξ, η, ζ) with unit vectors $\underset{\sim}{e}_\xi$, $\underset{\sim}{e}_\eta$, and $\underset{\sim}{e}_\zeta$, also has its origin at the geometrical center of the flat plate.

Obviously, any point in the two-dimensional space considered may be described by either coordinate system. Suppose $\underset{\sim}{r}$ is the vector from the origin to the point of interest. Then

$$\underset{\sim}{r} = \underset{\sim}{i}\, x + \underset{\sim}{j}\, y \tag{3}$$

or

$$\underset{\sim}{r} = \underset{\sim}{e}_{\xi}\, \xi + \underset{\sim}{e}_{\eta}\, \eta \tag{4}$$

Furthermore, it is easily seen

$$\begin{aligned} x &= \cos\theta\, \xi - \sin\theta\, \eta \\ y &= \sin\theta\, \xi + \cos\theta\, \eta \end{aligned} \tag{5a,b}$$

Conversely,

$$\begin{aligned} \xi &= \cos\theta\, x + \sin\theta\, y \\ \eta &= -\sin\theta\, x + \cos\theta\, y \end{aligned} \tag{6a,b}$$

If we decompose the integral into the plate portion of bound vorticity and the free vortex sheet portion, we can write instead of (2)

$$\begin{aligned} \underset{\sim}{V}(\underset{\sim}{r}_o) = {} & \int_{-1/2}^{1/2} \frac{\gamma(\xi)}{2\pi} \left\{ \frac{-\underset{\sim}{i}(y_o - \sin\theta\,\xi) + \underset{\sim}{j}(x_o - \cos\theta\,\xi)}{(x_o - \cos\theta\,\xi)^2 + (y_o - \sin\theta\,\xi)^2} \right\} d\xi \\ & + \int_{T.V.S.} \frac{\gamma(s)}{2\pi} \left\{ \frac{-\underset{\sim}{i}(y_o - y) + \underset{\sim}{j}(x_o - x)}{(x_o - x)^2 + (y_o - y)^2} \right\} ds + U_\infty \underset{\sim}{i} \end{aligned} \tag{7}$$

A useful result obtained from (7) is the velocity normal to the plate at some point ξ_o on the plate ($\eta_o = 0^{\pm}$). Thus

$$V_\eta(\xi_o, 0^{\pm}) = e_\eta \cdot V(\xi_o, 0^{\pm}) = \int_{-1/2}^{1/2} \frac{\gamma(\xi)}{2\pi} \frac{d\xi}{\xi_o - \xi} - U_\infty \sin\theta$$

$$+ \int_{T.V.S.} \frac{\gamma(s)}{2\pi} \left\{ \frac{\cos\theta\,(x_o - x) + \sin\theta\,(y_o - y)}{(x_o - x)^2 + (y_o - y)^2} \right\} ds \tag{8}$$

whereas the tangential velocity is

$$V_\xi(\xi_o, 0^{\pm}) = e_\xi \cdot V(\xi_o, 0^{\pm}) = \mp \frac{\gamma(\xi_o)}{2} + U_\infty \cos\theta$$

$$+ \int_{T.V.S.} \frac{\gamma(s)}{2\pi} \left\{ \frac{\sin\theta\,(x_o - x) - \cos\theta\,(y_o - y)}{(x_o - x)^2 + (y_o - y)^2} \right\} ds \tag{9}$$

Now, for a flat plate whose angular position is given as a function of time, $\theta(\tau)$, the nondimensional boundary condition to be satisfied is

$$V_\eta(\xi_o) = \xi_o \frac{d\theta}{d\tau} \quad ; \quad -\frac{1}{2} < \xi_o < \frac{1}{2} + \epsilon \tag{10}$$

$\theta(\tau)$ is an arbitrary input to the calculation. By extending the boundary condition an indefinitely small distance downstream of the trailing edge, the Kutta condition is satisfied; i.e., the trailing streamline is tangent

to the trailing edge. To determine the strength of shed vorticity, one makes use of Kelvin's theorem where

$$\frac{D}{Dt}\left\{ \int \gamma(s)\ ds \right\} = 0 \tag{11}$$

The limit of integration can include any portion of the free vortex sheet in a Lagrangian sense or it can include the airfoil and any contiguous portion of the vortex sheet.

III. THE FINITE DIFFERENCE SYSTEM OF EQUATIONS

The numerical scheme to arrive at solutions for the airfoil and free vortex strength distribution and the kinematics of the free vortex sheet will now be discussed. As a reference, the flow chart (see Appendix) will be instructive. The scheme is basically two-part. Knowing the strengths and positions of all vortices at $t = t_k$, the velocity field is calculated and the new location of the free vortices are calculated at $t = t_k + \Delta t$. The circulation strength of each free vortex is conserved. Then the unknown strengths of the bound vortices and most recently shed free vortex are calculated by satisfying boundary conditions as described by the finite difference counterpart of (8) and (10).

The vortex sheet representing the airfoil will be approximated by a finite number, N, of evenly distributed bound vortices. Bound vortex positions are indicated by circles in Fig. 1 (where, as an example, we choose N = 5). The distance between the vortices is

$$\Delta \xi = \frac{1}{N-1} \tag{12}$$

The matching points, ξ_{oi}, at which the boundary conditions, equation (10), are satisfied will be halfway between consecutive vortices, so that as $N \to \infty$, the normal velocity will approach the principal value of the integral given by (8). Thus, the finite number of evenly distributed bound vortices will be placed at

$$\xi_j = -0.5 + \frac{j-1}{N-1} \qquad (j = 1, 2, \ldots, N) \tag{13}$$

while the matching condition points will be located at

$$\xi_{oi} = -0.5 + \frac{i-0.5}{N-1} \qquad (i = 1, 2, \ldots, N-1) \tag{14}$$

The shed vortex sheet will be approximated as discrete doubly-infinite free point vortices shed from the trailing edge. A new vortex is shed at each increment in time and its strength is uniquely determined in satisfying the boundary conditions (equation 10). Once the strength of each vortex is determined, it will remain constant for all time.

To start the calculation, let us suppose that the vortex sheet distribution is known. The rate of change of position of the i^{th} free vortex during a time increment Δt is given by the finite difference counterpart of (7). Replacing the vortex sheet strength $\gamma\Delta s$ by Γ_j at $x, y = x_j, y_j$

$$V_{xi} = -\sum_{j=1}^{N} \frac{\gamma_j}{2\pi} \left\{ \frac{(y_i - \sin\theta\, \xi_j)}{(x_i - \cos\theta\, \xi_j)^2 + (y_i - \sin\theta\, \xi_j)^2} \right\} \Delta\xi \tag{15a}$$

$$- \sum_{\substack{j=1 \\ i \neq j}}^{M} \frac{\Gamma_j}{2\pi} \left\{ \frac{(y_i - y_j)}{(x_i - x_j)^2 + (y_i - y_j)^2} \right\} + U_\infty$$

$$V_{yi} = \sum_{j=1}^{N} \frac{\gamma_j}{2\pi} \left\{ \frac{(x_i - \cos\theta\, \xi_j)}{(x_i - \cos\theta\, \xi_j)^2 + (y_i - \sin\theta\, \xi_j)^2} \right\} \Delta\xi$$

$$+ \sum_{\substack{j=1 \\ i \neq j}}^{M} \frac{\Gamma_j}{2\pi} \left\{ \frac{(x_i - x_j)}{(x_i - x_j)^2 + (y_i - y_j)^2} \right\} \tag{15b}$$

Then the new position of each free vortex may be determined

$$x_{i+1}(t + \Delta t) = x_i(t) + V_{xi}\, \Delta t$$

$$y_{i+1}(t + \Delta t) = y_i(t) + V_{yi}\, \Delta t \tag{16a,b}$$

We have found it convenient to index by unity each free vortex. Thus the newly shed free vortex will always be identified by i = 1. Having established the positions of the free vortices according to (15) for $t = t_k + \Delta t$, combining (8) and (10) the boundary condition to be satisfied at that instant of time may be written in a finite difference sense as

$$\frac{1}{2\pi} \sum_{j=1}^{N} \frac{\gamma_j\, \Delta\xi}{\xi_j - \xi_{oi}} - \left[V_{f\, i1} \right]_{\eta} = \sum_{j=2}^{M} \left[V_{f\, ij} \right]_{\eta} \tag{17}$$

$$- U_\infty \sin\theta - \xi_{oi} \frac{d\theta}{d\tau}$$

with the right-hand side completely specified. $(V_{f\ ij})_\eta$ is the component of velocity normal to the surface at ξ_{oi} induced by the $j^{\underline{th}}$ free vortex. It is given by

$$\left[V_{f\ ij} \right]_\eta = \frac{\Gamma_j}{2\pi} \left\{ \frac{\xi_{oi} - (\cos\theta\, x_j + \sin\theta\, y_j)}{(\xi_{oi} \sin\theta - y_j)^2 + (\xi_{oi} \cos\theta - x_j)^2} \right\} \tag{18}$$

It may be shown that Kelvin's theorem (11) may be written as

$$\left\{ \sum_{j=1}^{N} \gamma_j \, \Delta\xi + \Gamma_1 \right\}_{t_{K+1} = t_K + \Delta t} = \left\{ \sum_{j=1}^{N} \gamma_j \, \Delta\xi \right\}_{t_K} \tag{19}$$

Equations (17), (18), and (19) may be combined and written as

$$\sum_{j=1}^{N+1} a_{ij} \, b_j = c_i \qquad (i = 1, 2, \ldots, N+1) \tag{20}$$

where

$$c_i = \sum_{j=2}^{M} \left[V_{f\ ij} \right]_\eta - U_\infty \sin\theta - \xi_{oi} \frac{d\theta}{dt} \tag{21}$$

$$c_{N+1} = \left\{ \sum_{j=1}^{N} \gamma_j \, \Delta\xi \right\}_{t = t_K} \tag{22}$$

and,

$$b_j = \gamma_j \qquad (j = 1, 2, \ldots, N) \tag{23}$$

$$b_{N+1} = \Gamma_1 \tag{24}$$

The coefficient matrix follows readily

$$a_{ij} = \frac{1}{2\pi} \frac{\Delta \xi}{\xi_j - \xi_{oi}} \qquad \left\{ \begin{array}{l} i = 1, 2, \ldots, N \\ j = 1, 2, \ldots, N \end{array} \right\} \tag{25}$$

$$a_{i\,N+1} = -\left[V_{f\,i1} \right]_{\eta} \qquad (i = 1, 2, \ldots, N) \tag{26}$$

Finally from (18)

$$a_{N+1\,j} = \Delta \xi \qquad (j = 1, 2, \ldots, N) \tag{27}$$

$$a_{N+1\,N+1} = 1.0 \tag{28}$$

Numerical representation of the boundary condition and Kelvin's theorem is now complete. Solution is obtained by inversion of the coefficient matrix. The equations were programmed in Fortran IV for the IBM 360-67 (see Appendix).

IV. CALCULATED RESULTS

The solution for the field about a flat plate in steady uniform flow is first considered. A comparison is made in Fig. (2) between the vortex strength distribution determined by the finite difference technique and that predicted by the exact integral (see, for example, reference [4]). Greater agreement is achieved by increasing N, the number of bound vortices defining the flat plate. This is, as noted previously, due to the fact that as $N \to \infty$, the solution from the finite difference technique approaches the principal value of (8).

The case of impulsive acceleration from rest was considered with $\theta = 0.1$ rad (5.73°). The resultant shed vortex sheet is depicted in Fig. (3) for $\tau = 2.0$ and $\tau = 3.0$. The vortex strength distribution on the plate approaches that of steady state asymptotically with time while the strength of successive shed vortices becomes negligible. Fig. (4) shows c_1 (normalized on angle of attack) vs. τ. As expected, it approaches the analytical value of 2π asymptotically.

Harmonic oscillation in pitch was then considered. $\theta(\tau)$ was defined such that

$$\theta(\tau) = \theta_o \sin \omega\tau$$

The important physical parameters of θ_o and ω were set equal to 0.1 and 0.1, 1.0, 2.0, and 3.0 respectively. The number of bound vortices defining the plate, N, was set to 11 and the maximum number of time steps taken was 150. $\Delta\tau$ was fixed so that the longitudinal spacing of shed vortices would be of the order of $\Delta\xi$.

Figs. (5-10) show the resultant shed vortex configurations for the specified frequencies at selected elapsed times. Since the wake is approximated by discrete free vortices, curves drawn sequentially through the points yield an approximation to the shed vortex sheet. As distance downstream increases, definition of the sheet becomes more difficult.

The short period frequency of a stable aircraft at cruise speed corresponds to a reduced frequency of the order of 0.1. As seen in Fig. (5), the resultant wake has negligible vertical deflection. This reaffirms the assumption that the wake is linear for most practical purposes in unsteady aerodynamics. Comparison of Figs. (6-10) demonstrates the increasing nonlinearity of the wake with increasing frequency.

An interpretation is made in Fig. (10) of the vortex sheet for $\omega = 3.0$ and $\tau = 10.0$. At a sufficient distance downstream, the wake rolls up periodically into concentrations of free vortices. The spacing ratio (that is, the ratio of the vertical displacement of the "centers of gravity" of cloisters of vortices of opposite sign to the horizontal distance between cloisters of like sign) is typically 0.07. The classical ratio predicted by von Karman [5] for a stable configuration of double infinite rows of point vortices of alternating sign is 0.281.

Coefficients of lift and drag were calculated by applying the Kutta-Joukowsky Law locally to each bound vortex defining the flat plate. Figs. (11, 12) present these values for $\omega = 3.0$ as a function of elapsed time τ. A typical value for c_l at $\tau = 0.50$ and $\theta = -5.71^{\circ}$ is .28.

The steady state value for this particular angle of attack is .61. c_l and c_d are periodic with time with negligible phase shift relative to the harmonic oscillation.

V. CONCLUSION AND DISCUSSION

The method developed herein yields good results for the unsteady, two-dimensional flow about a flat plate executing two separate nonlinear motions: impulsive acceleration and small amplitude oscillation about a spanwise axis. With the flow field specified, such significant quantities as stability derivatives are obtained readily.

In future efforts, bluff body flow, such as large amplitude oscillation or tumbling, with vortex sheet shedding from the trailing and leading edges, is contemplated. Preliminary investigations were made. Once again the potential flow integral equation representing the field velocity (1) is replaced by a finite difference counterpart. The boundary condition, (8) and (10), is extended an indefinitely small distance upstream of the leading edge and downstream of the trailing edge. (15, 17, 18, 19, 20) are modified accordingly. Kelvin's theorem determines the rate of vortex shedding and the two Kutta conditions establish uniqueness. Initial attempts to establish the flow field were unsuccessful. These attempts suggested that further refinement is needed in the numerical model. Consideration must be given vortices in close proximity (possibly in the form of the criterion proposed by Ham [2]). Furthermore, some modification may be required for the boundary condition at the extremities of the lifting surface.

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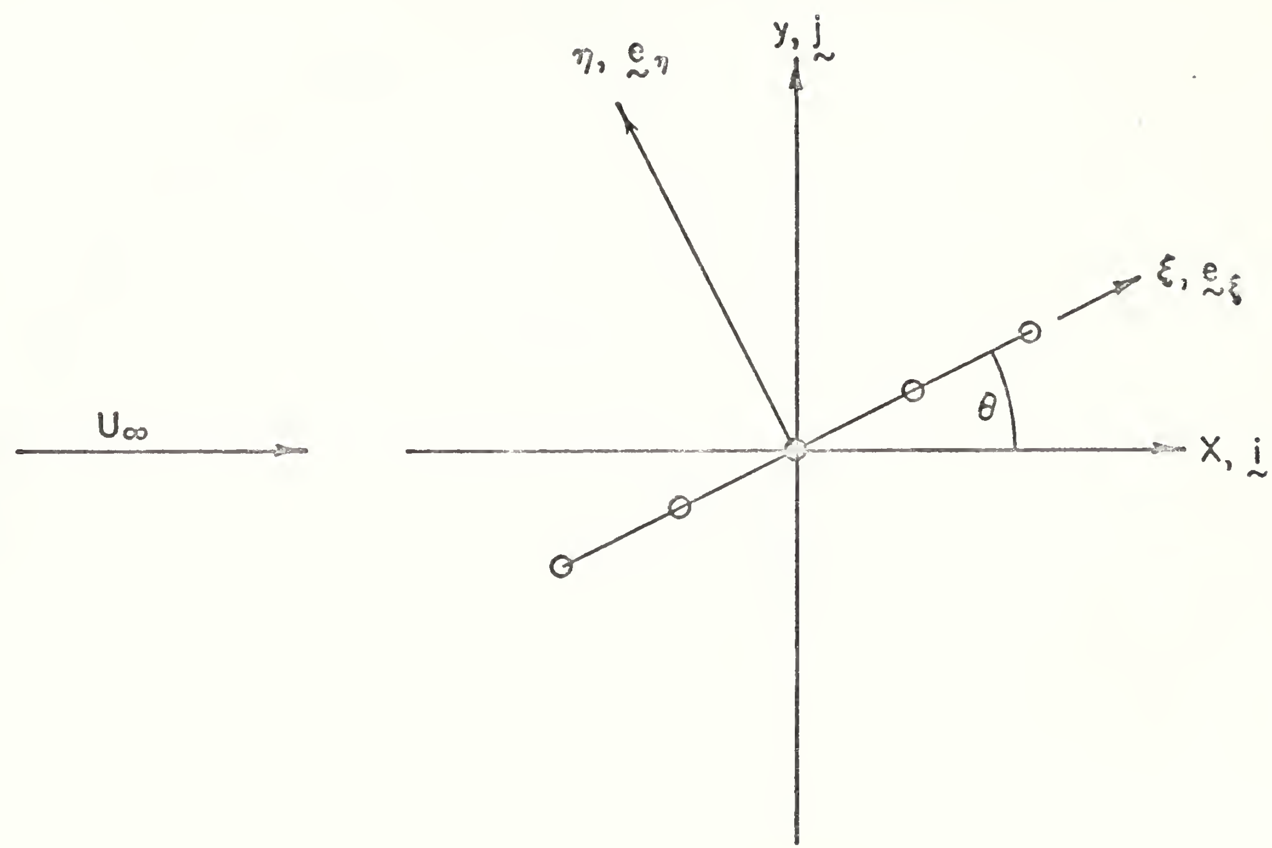

Figure 1. Coordinate Systems

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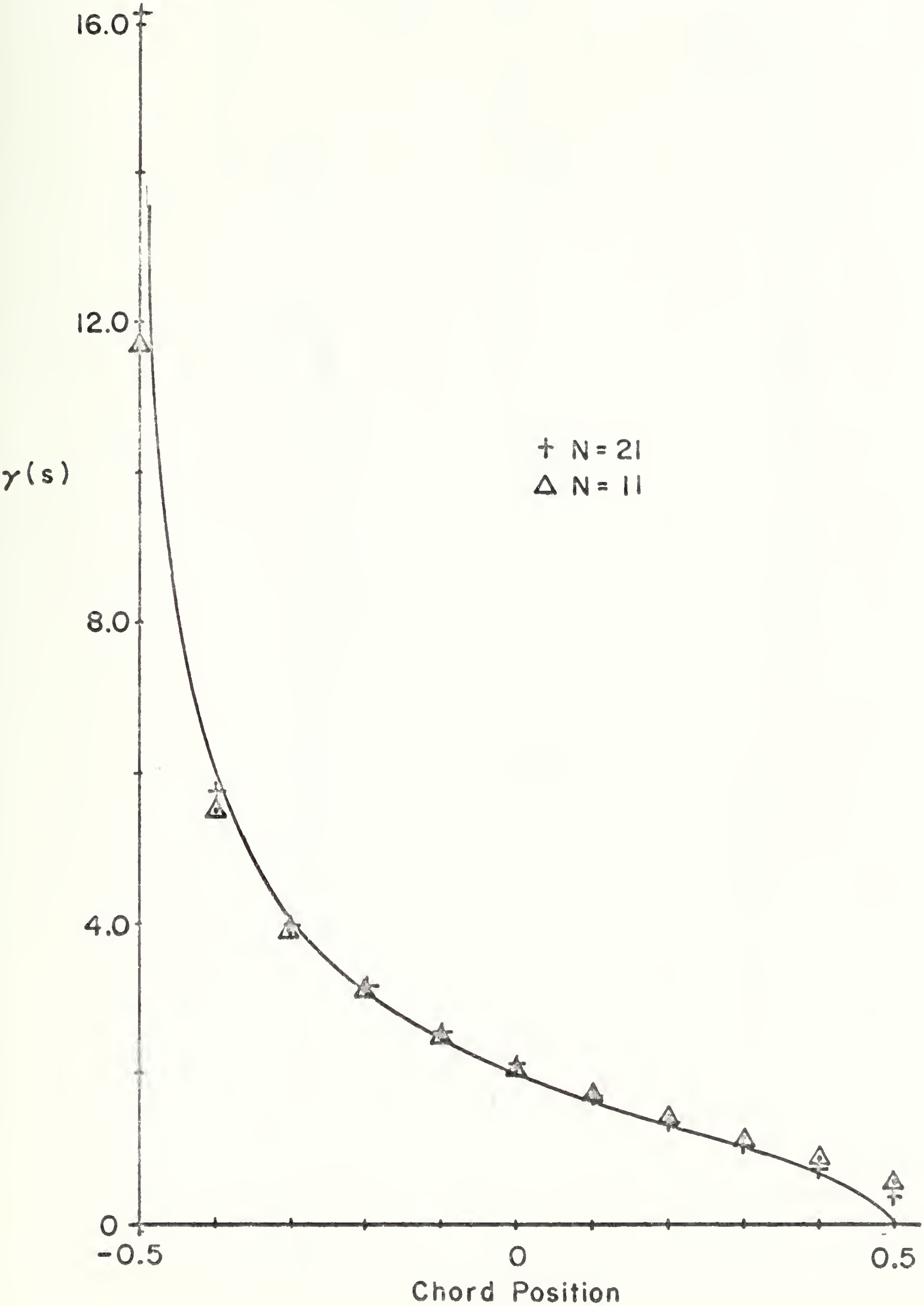

Figure 2. Steady State Numerical Solution Compared to Classical Flat Plate Theory.

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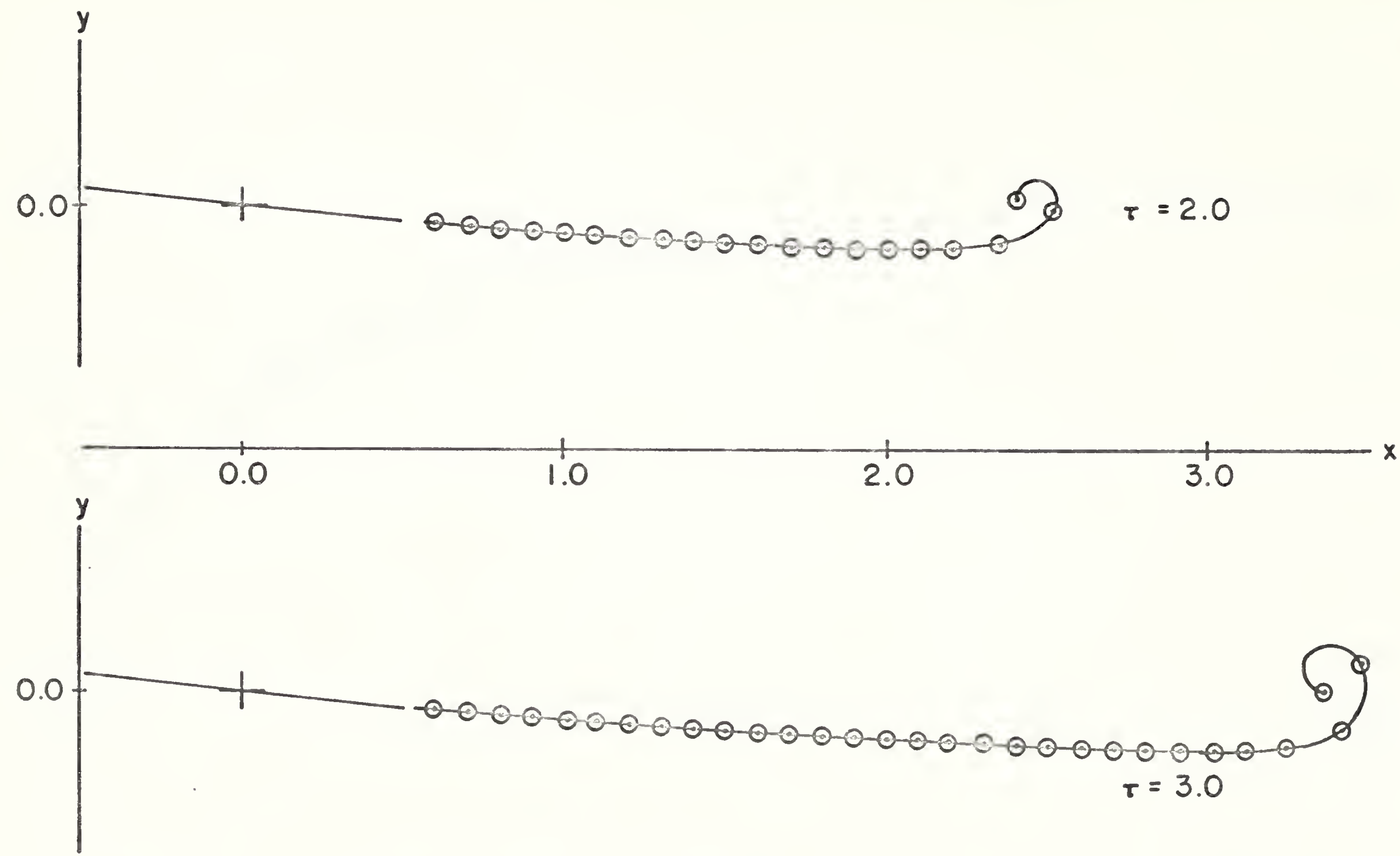

Figure 3. Vortex Sheet Configuration for Impulsively Started Flat Plate at Two Times.

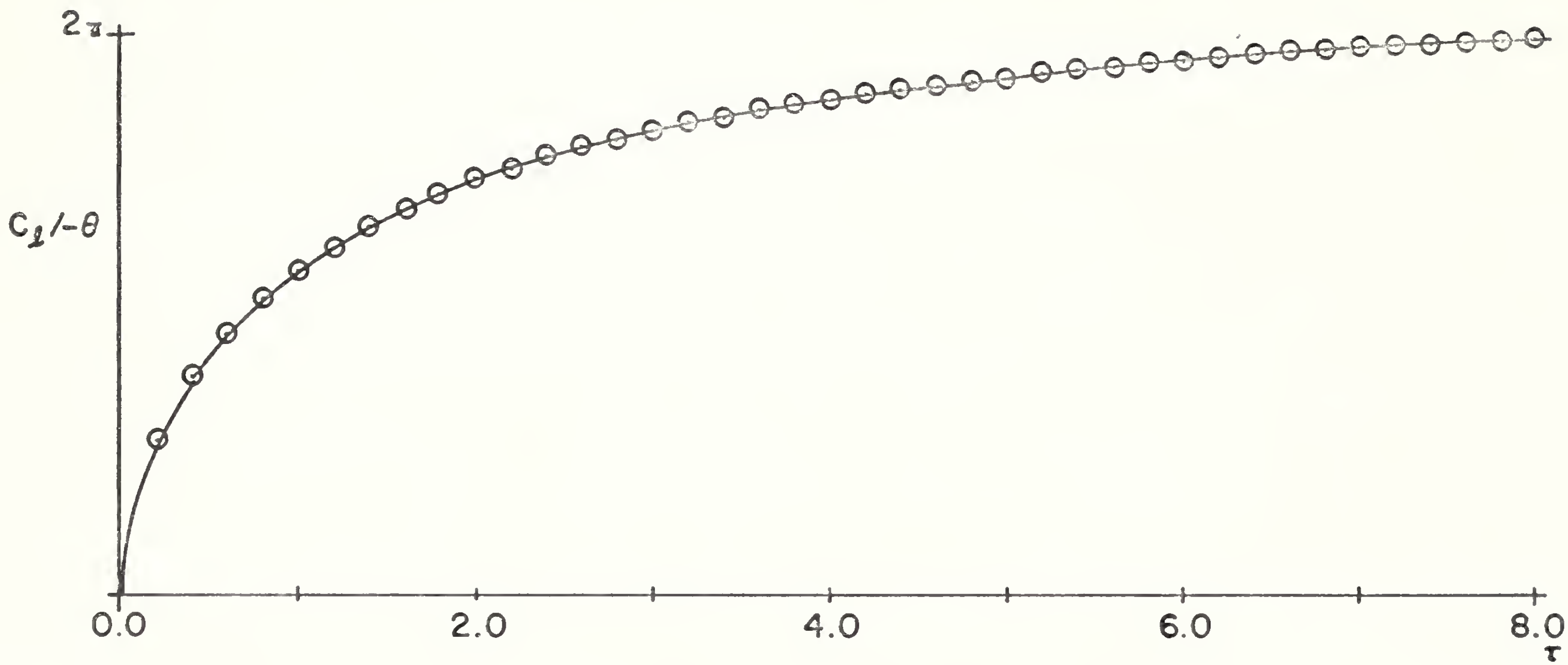

Figure 4. Coefficient of Lift (Normalized on Angle of Attack) as a Function of Time for the Impulsively Started Flat Plate.

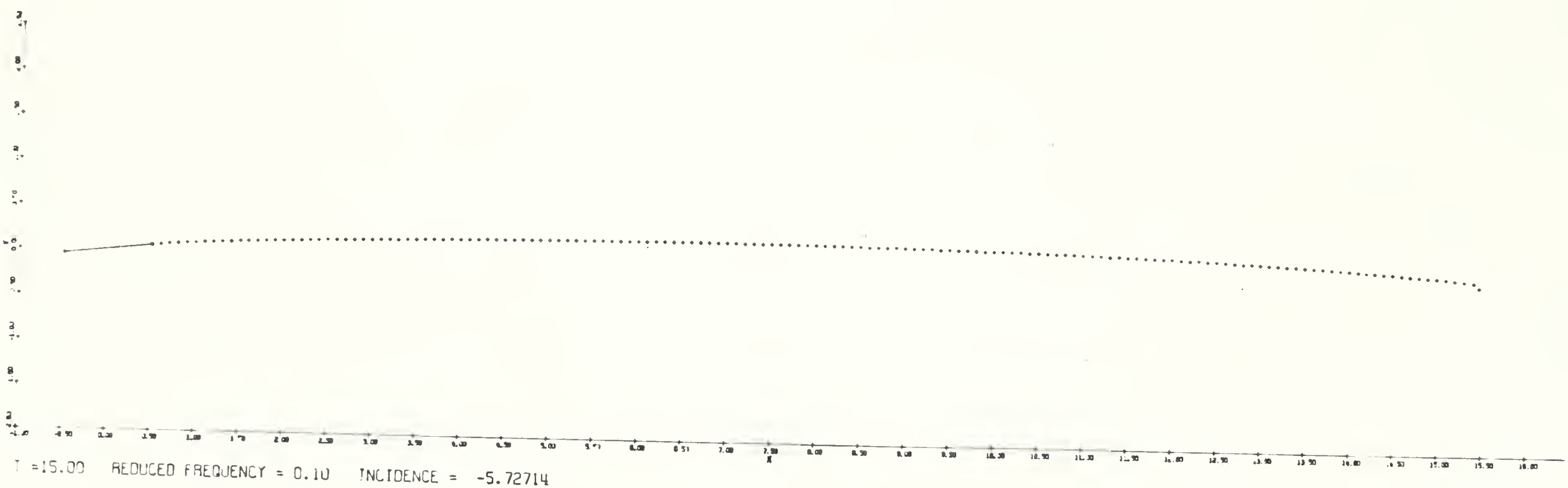

Figure 5. Vortex Sheet Configuration for Harmonically Oscillating Flat Plate.

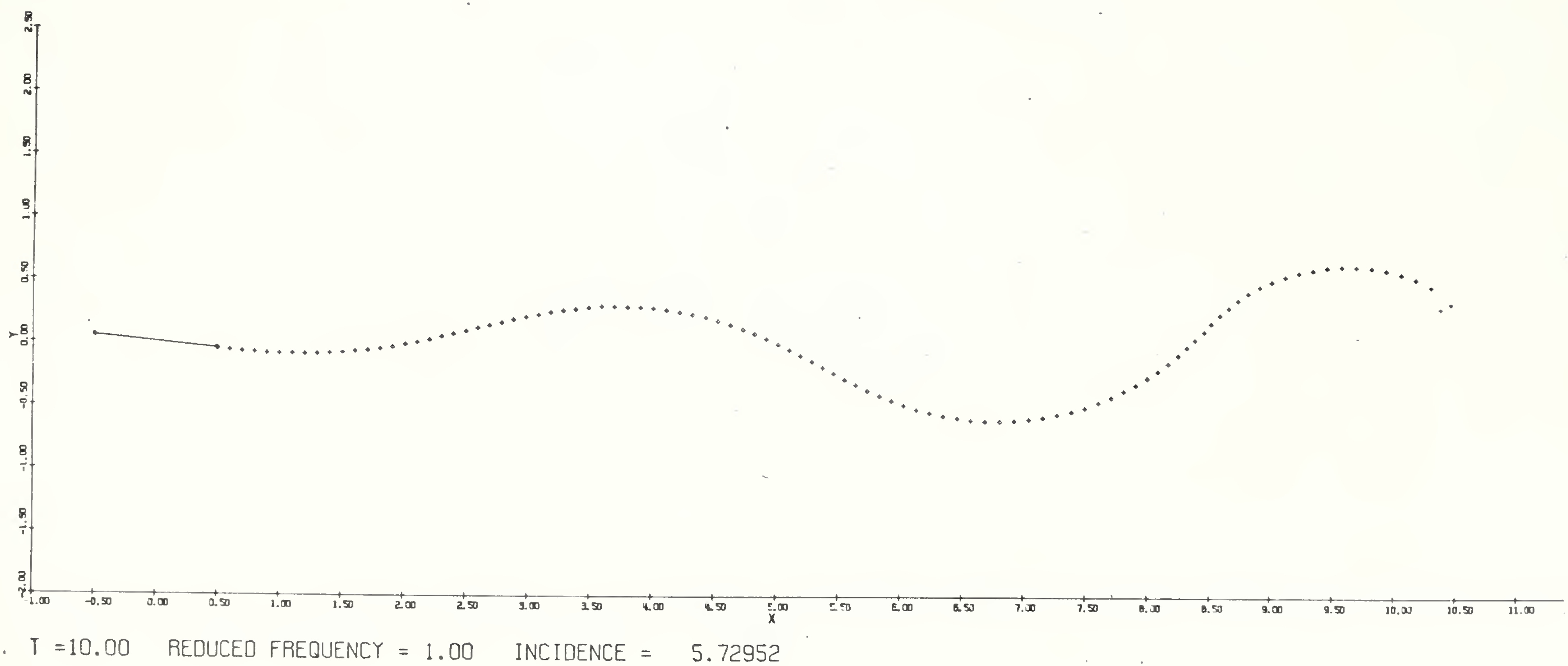

Figure 6. Vortex Sheet Configuration for Harmonically Oscillating Flat Plate

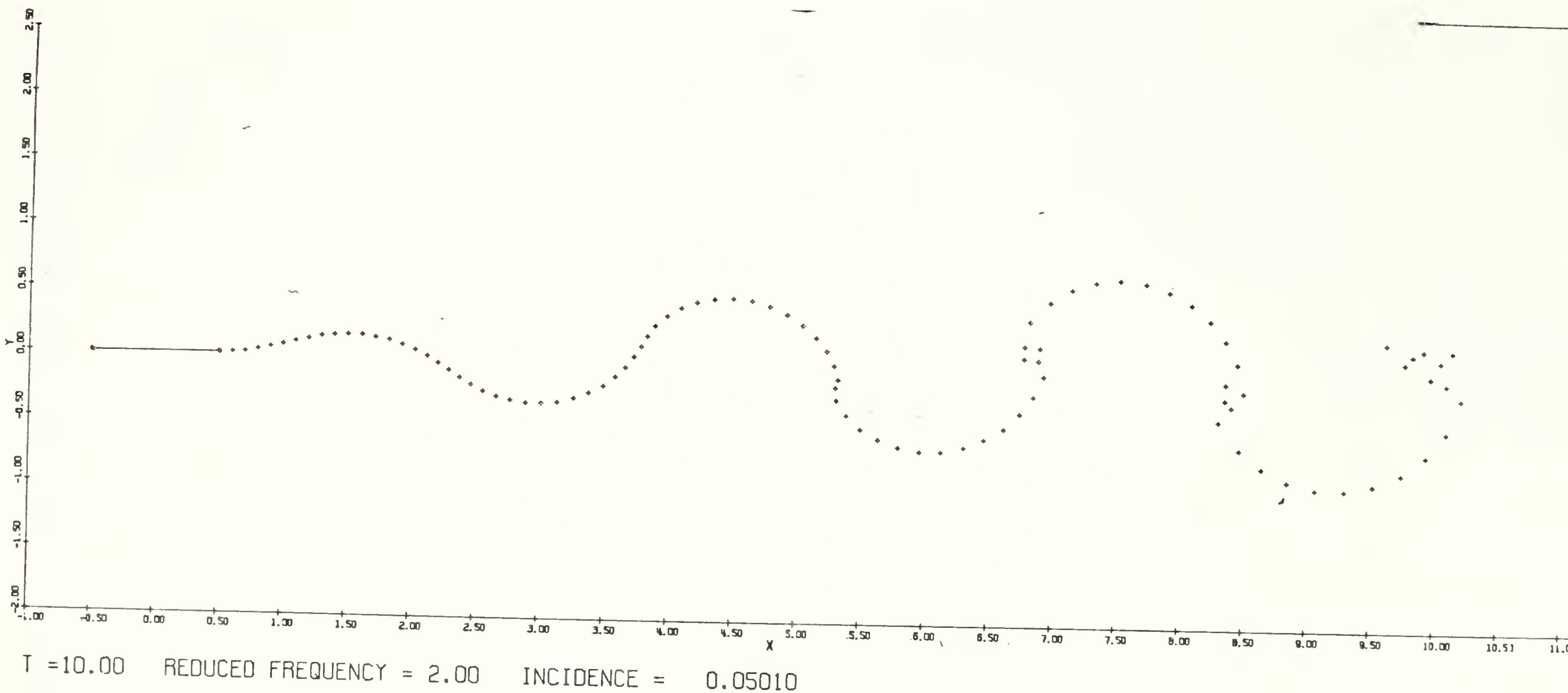

Figure 7. Vortex Sheet Configuration for Harmonically Oscillating Flat Plate

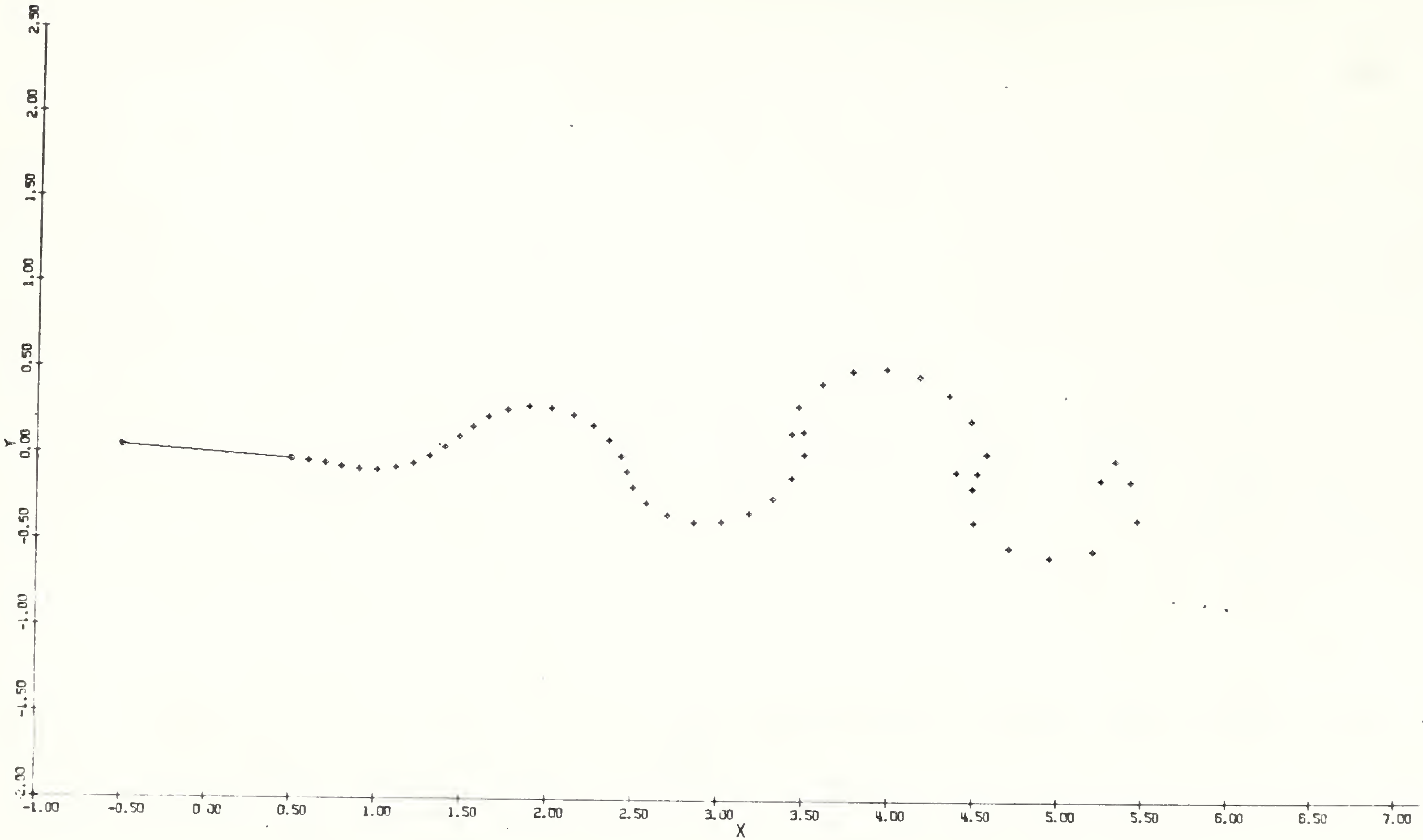

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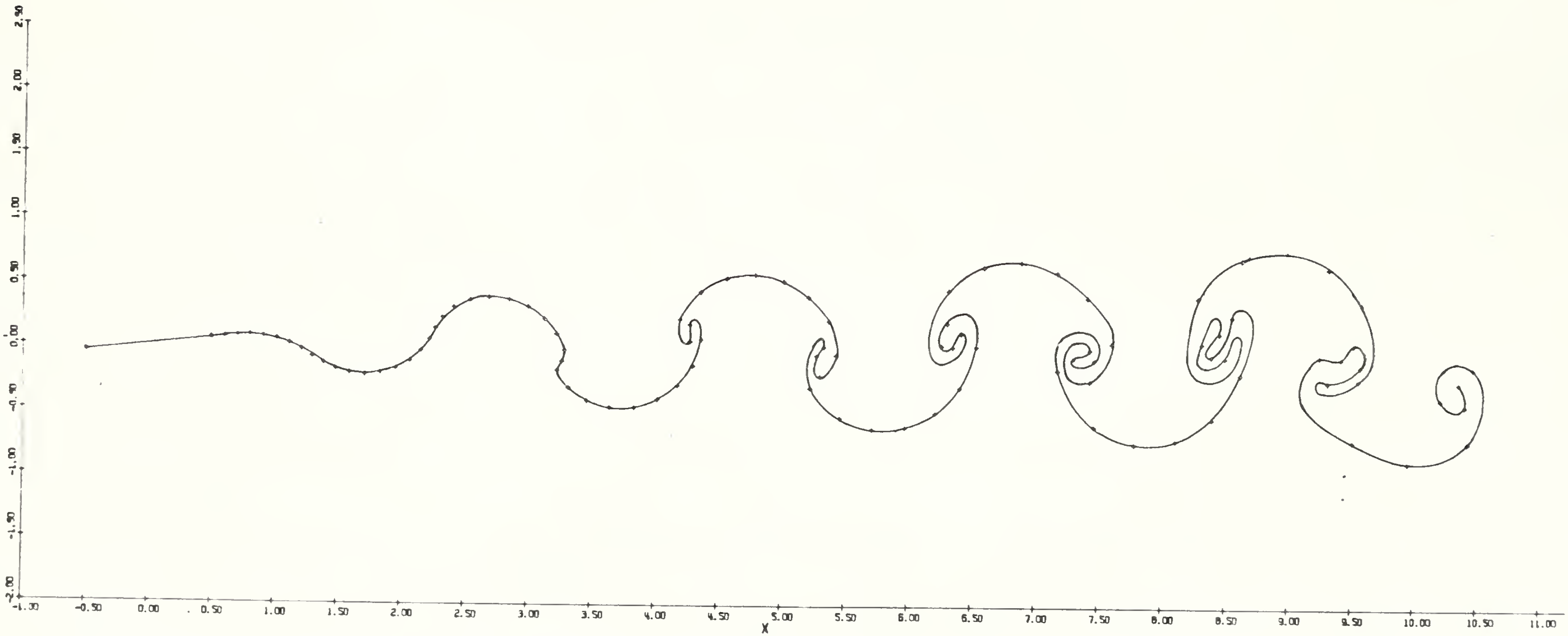
Figure 8. Vortex Sheet Configuration for Harmonically Oscillating Flat Plate



T =10.00 REDUCED FREQUENCY = 3.00 INCIDENCE = -5.72908

Figure 9. Vortex Sheet Configuration for Harmonically Oscillating Flat Plate

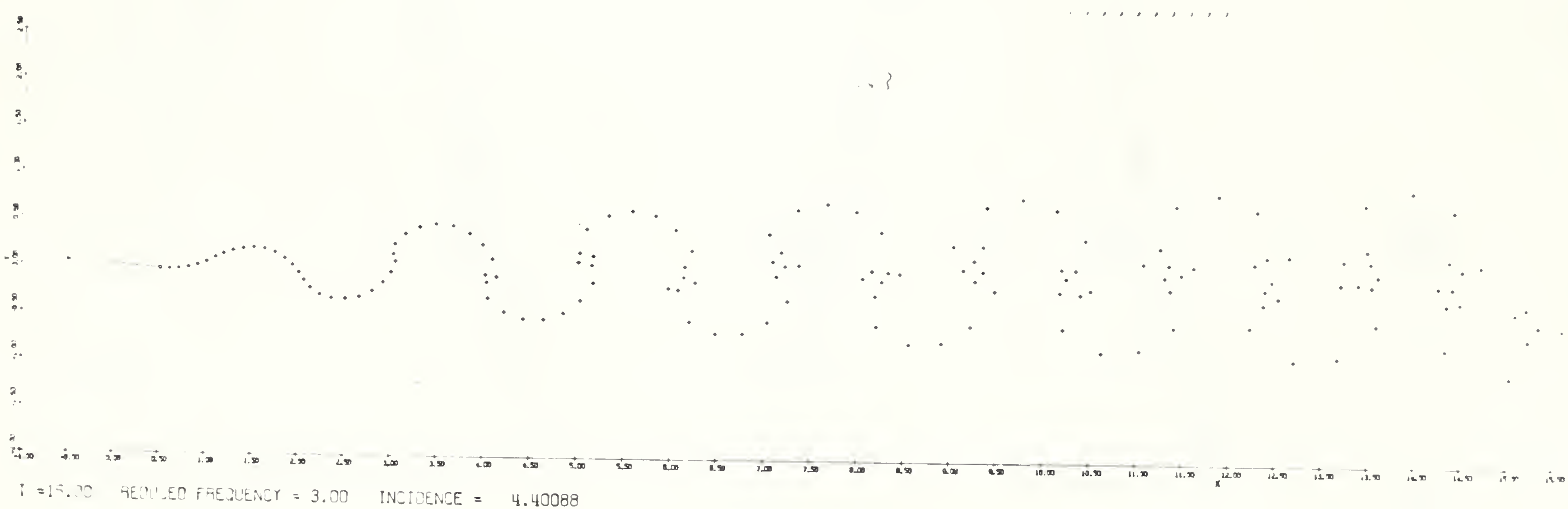


Figure 10. Vortex Sheet Configuration for Harmonically Oscillating Flat Plate

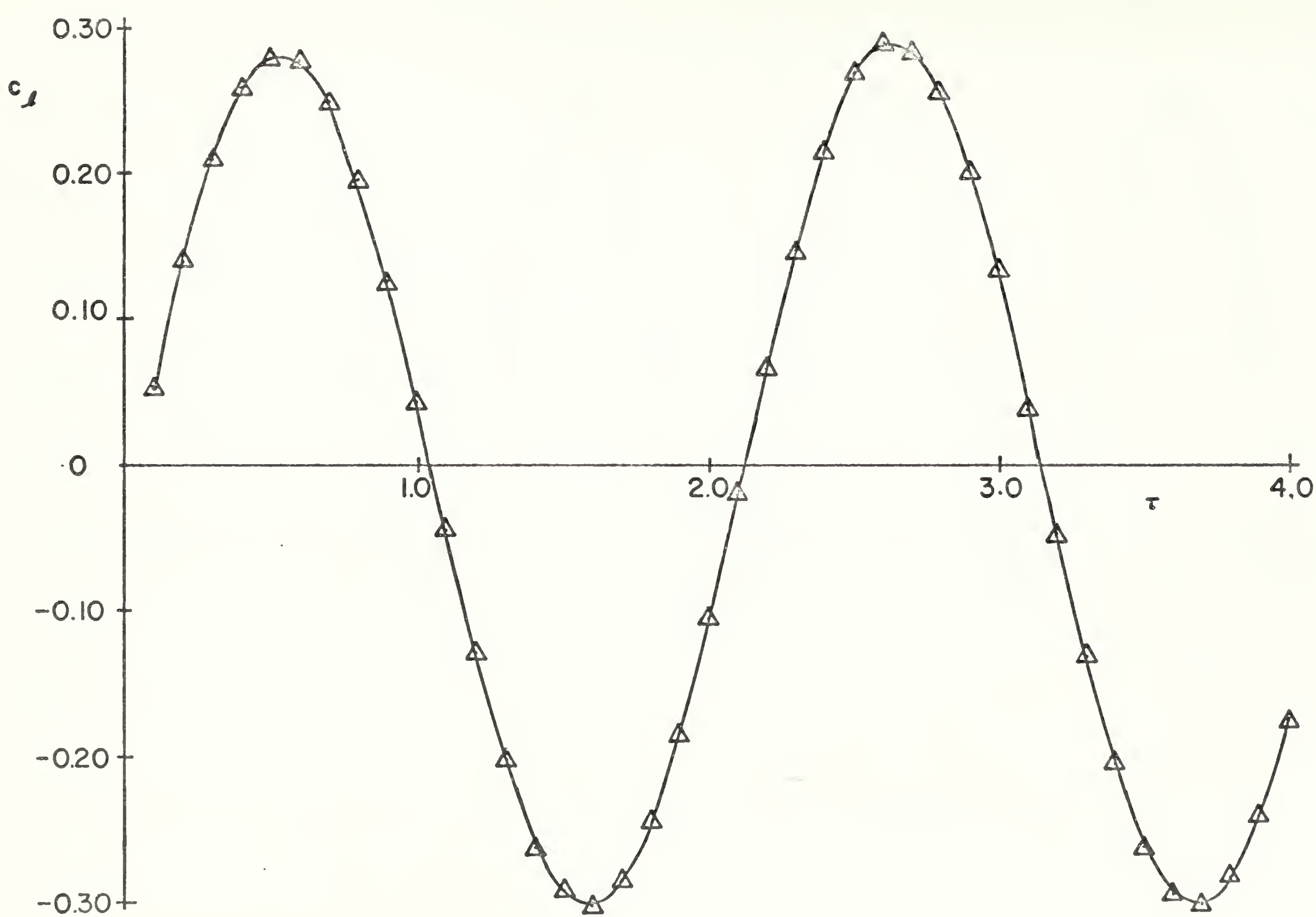


Figure 11. Coefficient of Lift as a Function of Time for Harmonically Oscillating Flat Plate With $\omega = 3.0$.

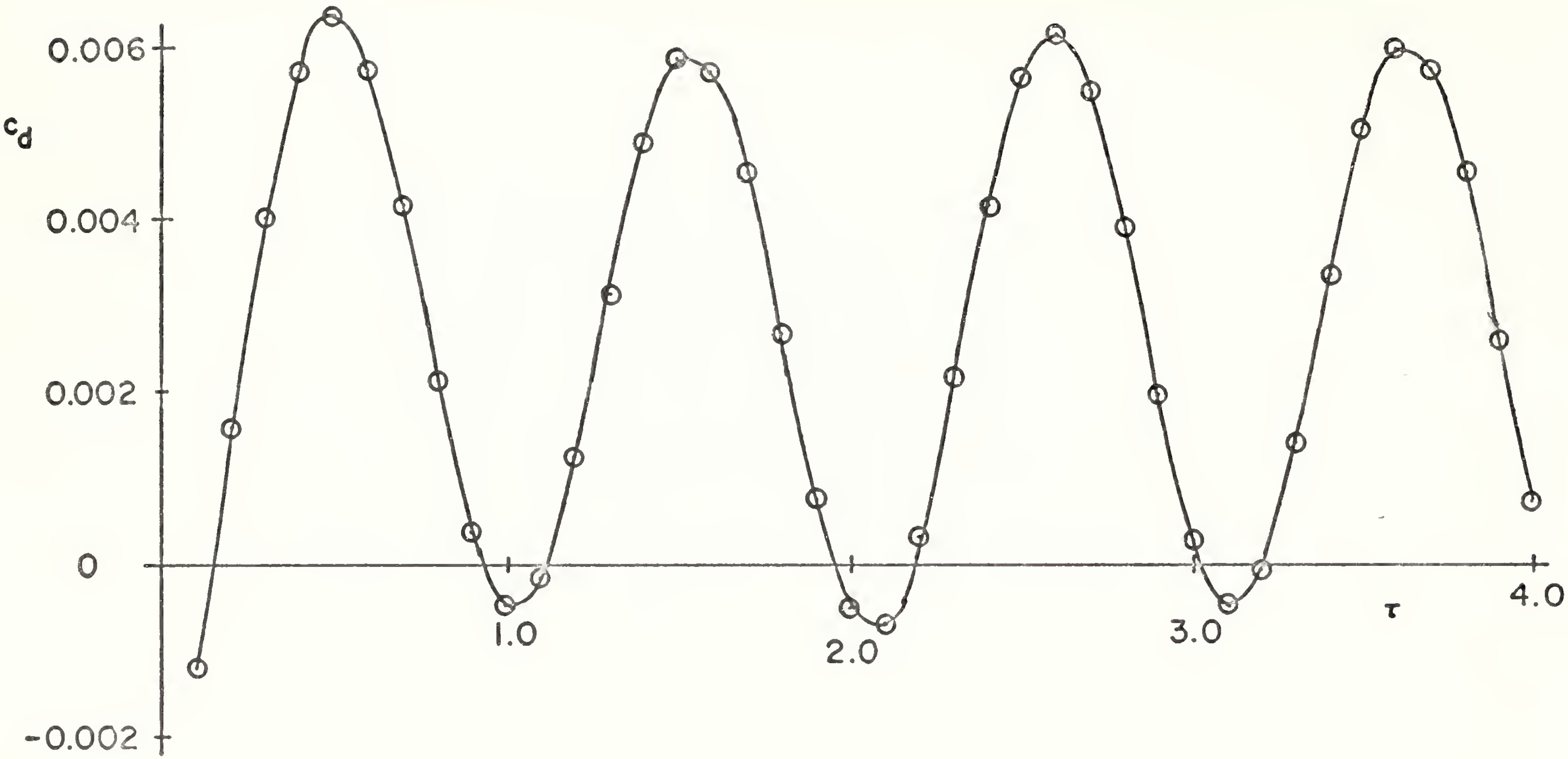
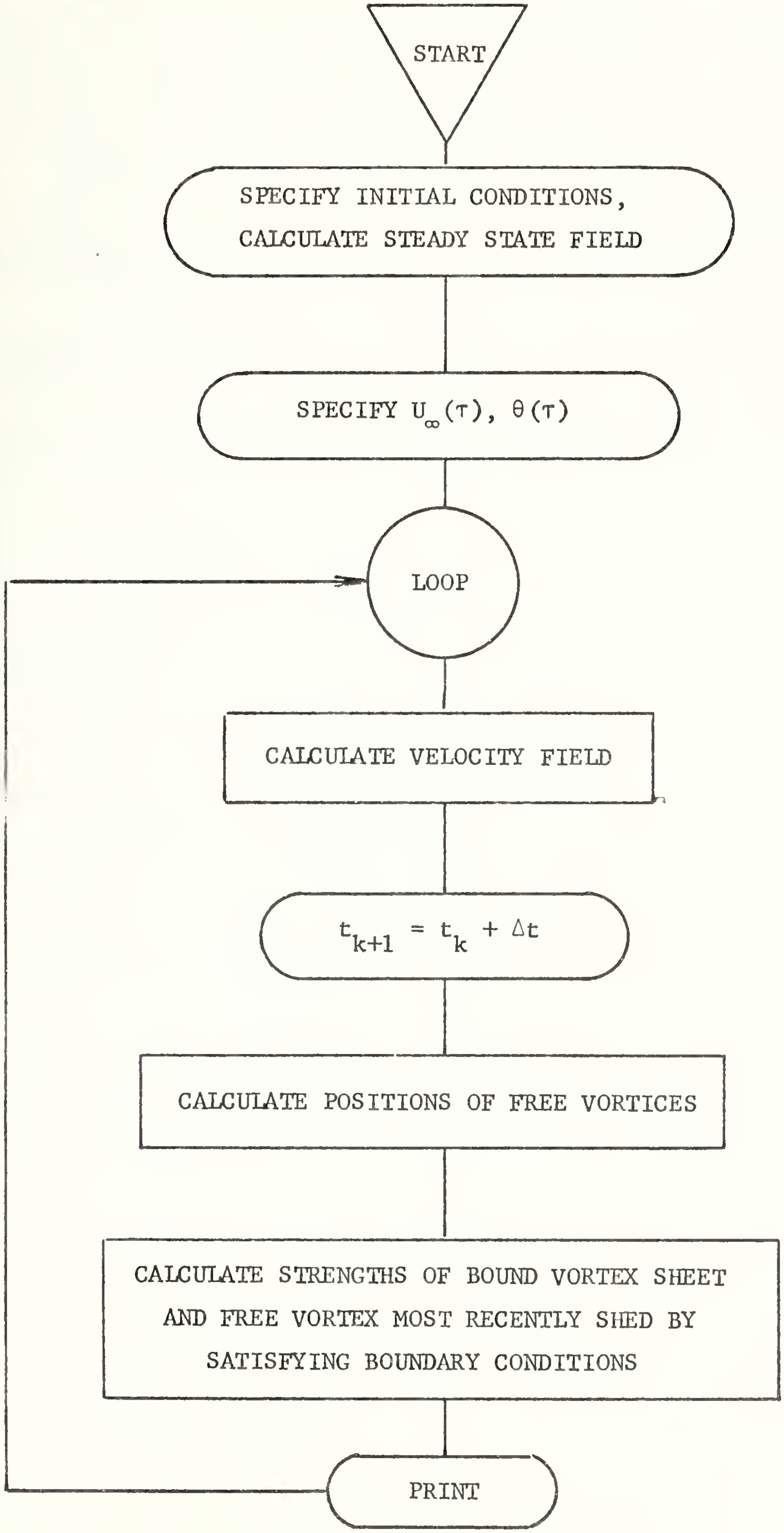


Figure 12. Coefficient of Drag as a Function of Time for Harmonically Oscillating Flat Plate With $\omega = 3.0$.

APPENDIX

OSCILLATING FLAT PLATE

COMPUTER PROGRAM



START
SPECIFY INITIAL CONDITIONS, CALCULATE STEADY STATE FIELD
SPECIFY $U_\infty(\tau)$, $\theta(\tau)$
LOOP
CALCULATE VELOCITY FIELD
$t_{k+1} = t_k + \Delta t$
CALCULATE POSITIONS OF FREE VORTICES
CALCULATE STRENGTHS OF BOUND VORTEX SHEET AND FREE VORTEX MOST RECENTLY SHED BY SATISFYING BOUNDARY CONDITIONS
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C
C     PUNCH DECK, OSCILLATING FLAT PLATE,
C
      REAL*4NXM,NYM
      DIMENSION E(11,11), F(11), NG(11), NP(11), H(11), Q(11)
      DIMENSION XB(11), YB(11), XF(200), YF(200), XM(11), YM(11)
      DIMENSION GAMMAF(200), X1M(11)
      DIMENSION A(12,12), B(12), C(12), L(12), NV(12), VNM(10)
      DIMENSION TGAM(200), ZX(200), ZY(200), X1B(11)
      PI = 6.28318
C
C     N IS THE NUMBER OF EVENLY DISTRIBUTED BOUND VORTICES ON THE LIFT-
C     ING SURFACE.  AN IS THE INCREMENTAL DISTANCE IN THE FINITE DIFFE-
C     RENCE SCHEME.
C
C     T IS THE ELAPSED TIME, TT IS THE INITIAL TIME AND DELT IS THE TIME
C     INCREMENT.
C
C     THETAZ IS THE AMPLITUDE OF OSCILLATION IN RADIANS WHILE FREQ IS
C     THE REDUCED FREQUENCY OF OSCILLATION.
C
      N=11
      AN = 1.0/FLOAT(N-1)
      N1 = N-1
      N2 = N+1
      THETAZ = 0.1
      FREQ = 0.1
      DELT = 0.1
      T = 0.0
      TT = -1.0
      TZ = T - TT
      DO 130 J = 1, N
      X1B(J) = -0.5 + AN * FLOAT(J-1)
      X1M(J) = -0.5 + AN * (0.5 + FLOAT(J-1))
  130 CONTINUE
C
C     THETA IS THE MODIFIED FREE STREAM ANGLE WHILE DTHETA IS THE CHANGE
C     IN THETA WITH RESPECT TO TIME.
C
      THETA = THETAZ * SIN(FREQ * TZ)
      DEGREE = THETA * 57.29583
      DTHETA = THETAZ * FREQ * COS(FREQ * TZ)
      COSTH = COS(THETA)
      SINTH = SIN(THETA)
      XTE = 0.5 * COSTH
      YTE = 0.5 * SINTH
      XXX = SQRT(XTE * XTE + YTE * YTE)
      NXM = -YTE/XXX
      NYM = XTE/XXX
      DO 140 J = 1,N
      XB(J) = X1B(J) * COSTH
      YB(J) = X1B(J) * SINTH
      XM(J) = X1M(J) * COSTH
      YM(J) = X1M(J) * SINTH
  140 CONTINUE
      DO 150 I = 1,N
      DO 150 J = 1,N
      E(I,J) = 1./(PI * (FLOAT(J-I) - 0.5))
 150  CONTINUE
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C
      DO 350 I = 1,N
      F(I) = NXM
 350  CONTINUE
      IROW = 1
      CALL MINV(E,N,D,NG,NP)
      CALL GMPRD(E,F,H,N,N,IROW)
      DO 60 I = 1, N
      B(I) = H(I)
   60 CONTINUE
C
C    THE INITIAL BOUND VORTEX STRENGTH  DISTRIBUTION IS SPECIFIED.
C
      CONTINUE
C
C     THE UNSTEADY FINITE DIFFERENCE SOLUTION FOLLOWS.
C
      DO 1000 M=1,150
      T = T + DELT
      TZ = T - TT
C
C     THE FREE VORTICES ARE ASSUMED SHED FROM THE TRAILING EDGE.
C
      XF(1) = XB(N)
      YF(1) = YB(N)
C
C     THE VELOCITY FIELD IS DETERMINED.
C
      DO 800 J = 1, M
C
C     VELOCITY INDUCED BY THE ITH FREE VORTEX ORIGINALLY SHED FROM THE
C     TRAILING EDGE ON THE JTH FREE VORTEX ORIGINALLY SHED FROM THE
C     TRAILING EDGE.
C
      VXTT = 0.0
      VYTT = 0.0
      IF(M.EQ.1) GO TO 501
C
C     GAMMAF(1) IS AS YET UNKNOWN.
C
      DO 500 I = 2,M
C
C     A VORTEX CANNOT ACT ON ITSELF.
C
      IF(I.EQ.J) GO TO 500
      X = XF(J) - XF(I)
      Y = YF(J) - YF(I)
      XX = X * X
      YY = Y * Y
      DENOM = PI * (XX + YY)
      VXTT = ((-Y * GAMMAF(I))/DENOM) + VXTT
      VYTT = (( X * GAMMAF(I))/DENOM) + VYTT
 500  CONTINUE
 501  CONTINUE
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C
C     VELOCITY INDUCED BY THE ITH BOUND VORTEX ON THE JTH FREE VORTEX
C     ORIGINALLY SHED FROM THE TRAILING EDGE.
C
      VXBT = 0.0
      VYBT = 0.0
C
C     THE BOUND VORTEX AT XB(N) IS EXCLUDED TO ARTIFICIALLY SUPPRESS
C     THE SINGULARITY.
C
      DO 600 I = 1,N
      IF(I.EQ.N.AND.J.EQ.1) GO TO 600
      X = XF(J) - XB(I)
      Y = YF(J) - YB(I)
      XX = X * X
      YY = Y * Y
      DENOM = PI * (XX + YY)
      VXBT = ((-Y * B(I) * AN)/DENOM) + VXBT
      VYBT = (( X * B(I) * AN)/DENOM) + VYBT
 600  CONTINUE
C
C     THE POSITIONS OF THE FREE VORTICES ARE DETERMINED FOR
C     T = T(K) + DELT.
C
      ZX(J) = (VXTT + VXBT) * DELT + DELT + XF(J)
      ZY(J) = ( VYTT + VYBT) * DELT + YF(J)
 800  CONTINUE
      DO 860 J = 1,M
      XF(J) = ZX(J)
      YF(J) = ZY(J)
  860 CONTINUE
      THETA = THETAZ * SIN(FREQ * TZ)
      DEGREE = THETA * 57.29583
      ALPHA = -(DEGREE)
      DTHETA = THETAZ * FREQ * COS(FREQ * TZ)
      COSTH = COS(THETA)
      SINTH = SIN(THETA)
      XTE = 0.5 * COSTH
      YTE = 0.5 * SINTH
      XLE = -XTE
      YLE = -YTE
      XXX = SQRT(XTE * XTE + YTE * YTE)
      NXM = -YTE/XXX
      NYM = XTE/XXX
      DO 230 J = 1,N
      XB(J) = X1B(J) * COSTH
      YB(J) = X1B(J) * SINTH
      XM(J) = X1M(J) * COSTH
      YM(J) = X1M(J) * SINTH
  230 CONTINUE
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C
C     CALCULATING THE COMPONENTS OF THE ROW VECTOR, C(I).
C
C     EACH ELEMENT OF THE  ROW   VECTOR IS, AT A PARTICULAR MATCHING
C     CONDITION POINT, AN EXPRESSION OF THE NORMAL COMPONENTS OF FREE
C     STREAM VELOCITY AND VELOCITY INDUCED BY FREE VORTICIES (EXCEPT
C     THOSE MOST RECENTLY SHED) PLUS THAT COMPONENT DUE TO ROTATION OF
C     THE LIFTING SURFACE.
C
      DO 200 I = 1,N
C
C     CALCULATING THE INDUCED VELOCITY AT THE ITH MATCHING CONDITION
C     POINT DUE TO THE JTH FREE VORTEX ORIGINALLY SHED FROM THE TRAILING
C     EDGE, WITH THE EXCEPTION OF THAT ONE MOST RECENTLY SHED.
C
      VX = 0.0
      VY = 0.0
      IF (M.EQ.1) GO TO 101
      DO 100 J = 2, M
      X = XM(I) - XF(J)
      Y = YM(I) - YF(J)
      XX = X * X
      YY = Y * Y
      DENOM = PI * (XX + YY)
      VX = ((-Y * GAMMAF(J))/DENOM) + VX
      VY = (( X * GAMMAF(J))/DENOM) + VY
 100  CONTINUE
  101 CONTINUE
      VNM(I) = (VX * NXM + VY * NYM)
      C(I) = NXM + VNM(I) - (XIM(I) * DTHETA)
 200  CONTINUE
C
C     COMPONENT DUE TO KELVIN'S THEOREM.
C
      SUMGAM = 0.0
      DO 620 I = 1,N
      SUMGAM = B(I) * AN + SUMGAM
  620 CONTINUE
      C(N2) = SUMGAM
C
C     CONSTRUCTING THE COEFFICIENT MATRIX.
C
C     THOSE ELEMENTS DUE TO THE VELOCITY INDUCED BY THE JTH BOUND VORTEX
C     AT THE JTH MATCHING CONDITION POINT.
C
      DO 210 I = 1,N
      DO 110 J = 1, N
      A(I,J) = 1./(PI * (FLOAT(J-I) - 0.5))
 110  CONTINUE
 210  CONTINUE
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C
C     THOSE ELEMENTS DUE TO THE EFFECT OF THE FREE VORTEX MOST RECENTLY
C     SHED FROM THE TRAILING EDGE ON THE ITH MATCHING CONDITION POINT.
C
      DO 530 I = 1,N
      X = XM(I) - XF(1)
      Y = YM(I) - YF(1)
      XX = X * X
      YY = Y * Y
      DENOM = PI * (XX + YY)
      VX = -Y/DENOM
      VY = X/DENOM
      A(I,N+1) = -(NXM * VX + NYM * VY)
  530 CONTINUE
C
C     THOSE ELEMENTS DUE TO CONSIDERATION OF KELVIN'S THEOREM.
C
      DO 550 J =1,N
      A(N2,J) = AN
  550 CONTINUE
      A(N2,N2) = 1.
C
C     INVERSION OF A(I,J).
C
      CALL MINV(A,N2,D,L,NV)
C
C     INVERSE(A(I,J)) * C(I) = B(J)
C
      CALL GMPRD(A,C,B,N2,N2,IROW)
      GAMMAF(1) = B(N2)
      IF(M.EQ.150) GO TO 999
      GO TO 699
  999 CONTINUE
      WRITE(7,20) T, FREQ, ALPHA
   20 FORMAT(2F5.2, F10.5)
      WRITE(7,25) XLE, YLE
   25 FORMAT(2F10.5)
      WRITE(7,26) XTE, YTE
   26 FORMAT(2F10.5)
      WRITE(7,21) (XF(J), YF(J), J=1,M)
   21 FORMAT(2F10.5)
      WRITE(7,22)
   22 FORMAT('1000.')
  699 CONTINUE
      DO 700 I = 1,M
      TGAM(I) = GAMMAF(I)
      ZX(I) = XF(I)
      ZY(I) = YF(I)
  700 CONTINUE
      DO 710 I = 1,M
      GAMMAF(I+1) = TGAM(I)
      XF(I+1) = ZX(I)
      YF(I+1) = ZY(I)
  710 CONTINUE
 1000 CONTINUE
      STOP
      END
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